Matrix Model, Noncommutative Gauge Theory and the Tachyon Potential

Gautam Mandal and Spenta R. Wadia

Department of Theoretical Physics,

Tata Institute of Fundamental Research,

Homi Bhabha Road, Mumbai 400 005, INDIA.

email: mandal,wadia@theory.tifr.res.in

ABSTRACT

The $D2$ brane-anti-$D2$brane system is described in the framework of BFSS Matrix model and noncommutative (NC) gauge theory. The physical spectrum of fields is found by appropriate gauge fixing. The exact tachyon potential is computed in terms of these variables and an exact description of tachyon condensation provided. We exhibit multiple vortex production with increasing topological charge and interpret this as gradual conversion of the brane-antibrane system to $D0$ branes. The entire analysis is carried out using the known hamiltonian of the Matrix model, which is equivalent to the hamiltonian of the NC gauge theory. We identify the supersymmetric ground state of this hamiltonian with the tachyonic vacuum; Sen's conjecture about the latter follows simply from this identification. We also find two types of closed string excitations, solitonic (*a la* Dijkgraaf, Verlinde and Verlinde) as well as perturbative, around the tachyonic vacuum.

1 Introduction

Over the last couple of years open string theory on unstable D-branes has been studied extensively to describe (a) lower dimensional D-branes as solitons [1]-[21], (b) the closed string vacuum as the tachyonic vacuum[1] [22]-[38] and also (c) closed string excitations [39]-[43] around the tachyonic vacuum. Although the initial discussions were in the framework of first-quantized or second-quantized open string theories, a number of recent papers have used the framework of noncommutative gauge theories [44]-[53] which are known [44, 45] to describe open strings on a D-brane in the presence of a constant $B_{\rm NS}$ or a constant magnetic field. Solitonic D-branes in this language are discussed in [15]-[21], the tachyonic vacuum in [29, 30, 49, 31, 32, 33], whereas closed string excitations around the tachyonic vacuum in the NC framework are discussed in [16, 42].

Unstable D-brane configurations which involve the "wrong" Dp-brane (p odd for type IIA, even for IIB) are characterised by a *real* tachyon. In the NC gauge theory or its equivalent Matrix model formulation, such theories are mostly studied using a DBI-like action [34, 35] (or certain truncations from open string field theories) that assumes a certain dependence on the tachyon and gauge and matter fields on the brane. At the moment there is no first principles derivation of this action. Although there is considerable support for the basic features of this action, there are subtleties regarding the choice of variables and the form of the action especially around the tachyonic vacuum [30].

The situation with a brane-antibrane pair is somewhat different. Here the tachyon is complex, which can, once again, decay [1] to lower dimensional branes (solitons) or the vacuum (global minimum of the potential). The important difference from the case of the real tachyon, from the point of view of NC gauge theory or its equivalent Matrix formulation, is that this system is directly describable [54, 55, 56] using the well-known action/hamiltonian of the BFSS Matrix model [57] around a known background. This gives us a well-defined hamiltonian framework to discuss various features of this theory including the tachyonic vacuum. Since M-theory, and therefore Matrix model, in principle contains all the states of string theory, the brane-antibrane system offers an excellent testing ground to compare with the derivation of the same states, including the vacuum,

---

[1]The phrase "tachyonic vacuum" refers to the global minimum of the tachyon potential; there are no open string tachyonic directions in this vacuum.

in open string field theory.

In this paper, we will address the issue of tachyon condensation on a brane-antibrane system in the framework of noncommutative gauge theory. In Sections 2 and 3, we consider the complex tachyon of the $D2 - \overline{D2}$ system in the context of the BFSS Matrix model using the remarkable connection of the matrix model with noncommutative gauge theory. In this framework the $D2 - \overline{D2}$ system appears as a classical solution of the noncommutative $U_\infty(2)^2$ gauge theory in $2 + 1$ dimensions. We discuss the Hamiltonian formulation of this theory in a unitary gauge to identify the $U_\infty(1) \times U_\infty(1)$ gauge fields and the tachyons. We present an exact expression for the potential energy and present an explicit vacuum solution. In Section 4, we further gauge fix the $U_\infty(1) \times U_\infty(1)$ symmetry to exhibit the physical degrees of freedom: a complex tachyon field and two hermitian fields (one each for the two photons). We exhibit the potential in terms of the physical variables. In Section 5, we describe a sequence of solutions in which an increasingly large number of vortices are produced on the brane-antibrane pair to partly reduce the energy of the initial unstable system. We show, by explicit computation of the topological charge of these vortices, that these correspond to $D0$ branes; these facts are further verified by translating the solution back to the Matrix model description. In this way, the vacuum consists of a large number of vortices ($D0$ branes) cancelling all the energy of the brane-antibrane system, in conformity with Sen's conjecture [2, 3]. In Sec. 6 we find two classes of closed string states around the tachyonic vacuum: solitonic ones which are the analogs of the Matrix string of [60], and perturbative flux tube-like solutions analogous to the ones discussed in [16] for the case of the real tachyon.

While this work was in progress, we received [32] which overlaps with some aspects of the present paper. See also [33].

---

[2]We will use the notation $U_\infty(2)$ to imply $U(\infty) \otimes U(2)$ which is the gauge group of $U(2)$ noncommutative gauge theory.

3

2 Matrix model and noncommutative gauge theory

The BFSS matrix model [57] is characterised by the following Hamiltonian[3] (in $2\pi l_s^2 = 2\pi\alpha' = 1$ units)

$$H = \frac{\sqrt{2\pi}}{g_s} \, \text{Tr} \left(\frac{1}{2} \sum_{M=1}^{9} (\dot{X}^M)^2 - \frac{1}{4} \sum_{M,N} [X^M, X^N]^2 + \Psi^T \gamma_M [\Psi, X^M] \right), \qquad (1)$$

and the Gauss-law constraint

$$[X^M, \dot{X}^M] + i\{\Psi^T, \Psi\} = 0 \qquad (2)$$

The $X^M, M = 1, .., 9$ are $N \times N$ matrices (N representing the number of $D0$-branes) and g_s is the string coupling constant.

A classical configuration of k coincident $D2p$ branes is described by [54]

$$
\begin{aligned}
X_{\text{cl}}^a &= x^a \otimes \mathbf{1}_{k \times k}, \ a = 1, 2, \ldots, 2p, \\
X_{\text{cl}}^i &= 0. \ i = 2p+1, \ldots, 9
\end{aligned}
\qquad (3)
$$

where x^a satisfy the Heisenberg algebra:

$$[x^a, x^b] = i\theta^{ab} \qquad (4)$$

In the case of the $D2$ brane, we will take

$$\theta^{ab} = \theta\epsilon^{ab}, \ \epsilon^{12} = 1 \qquad (5)$$

so that (4) simply becomes

$$[x^1, x^2] = i\theta \qquad (6)$$

The Heisenberg algebra (6) can be represented in a one-particle Hilbert space $\mathcal{H}$ where x^1, x^2 are identified with the position and the momentum respectively, θ playing the role of $\hbar$.

As is well-known, (6) does not have finite dimensional representations. In the BFSS framework it is, however, useful (as we will see in Section 5) often to regard the size of the

---

[3]The velocity $\dot{X}^M$, in the hamiltonian framework, should be regarded as $\dot{X}^M = g_s \, \Pi^M$.

matrices X^M to be large but finite (it represents the total $D0$ charge or the total "longitudinal" momentum in the M-theory direction). One such (approximate) N dimensional representation is constructed through $N \times N$ matrices U and V, satisfying

$$U^N = V^N = 1, \ UV = e^{2\pi i/N} VU \tag{7}$$

The matrices U, V can be explicitly constructed for any N. Using these, one defines x^1, x^2 by

$$U = e^{ix^1}, \ V = e^{ix^2} \tag{8}$$

These satisfy

$$[x^1, x^2] \approx 2\pi i/N \tag{9}$$

the approximation becoming exact in the limit $N \to \infty$. In this way, the matrices in (3) for $p = 1$ will be $kN \times kN$, viewed as adjoints of $U(k) \otimes U(N) \equiv U_N(k)$. Having said this, we will now go back to infinite-dimensional representations of (4) or (6), represented by operators in the Hilbert space $\mathcal{H}$. We will denote the group of transformations of such operators as $U(k) \otimes U(\infty) \equiv U_\infty(k)$.

We will parameterize the bosonic fluctuations of the X^M around (3) as [54, 49]

$$\begin{aligned} X^a &= x^a \otimes \mathbf{1}_{k \times k} + \theta^{ab} \mathcal{A}_b(x) \\ X^j &= \phi^j \end{aligned} \tag{10}$$

Here $\mathcal{A}_a$ is a $k \times k$ hermitian matrix of hermitian operators $\mathcal{A}_{a,mn}(x), n, m = 1, \ldots, k$ (similarly for ϕ^j). The fundamental observation, made in early Matrix model literature [54, 61] and emphasized recently in [49, 52], is that the group of unitary transformations on the matrices gets represented as $U_\infty(k)$ gauge transformations on $\mathcal{A}_{a,mn}(x)$:

$$\mathcal{A}_a \to \mathcal{U}^\dagger \mathcal{A}_a \mathcal{U} + i\mathcal{U}^\dagger \partial_a \mathcal{U} \tag{11}$$

where $\partial_a = -i(\theta^{-1})_{ab}[x_b, .]$ is the derivative (see, e.g., [48]) operator.

The matrix model Hamiltonian (1) and the Gauss law (2) can now be written as those of a $U(k)$ noncommutative gauge theory:

$$\begin{aligned} H &= \frac{\sqrt{2\pi}}{g_s} \text{Tr} \left[\frac{1}{2} G^{ab} \dot{\mathcal{A}}_a \dot{\mathcal{A}}_b + \frac{1}{4} G^{ac} G^{bd} (\mathcal{F}_{ab} - (\theta^{-1})_{ab})(\mathcal{F}_{cd} - (\theta^{-1})_{cd}) \right. \\ &\quad + \left. \frac{1}{2}(\dot{\phi}^i)^2 + \frac{1}{2} G^{ab} \mathcal{D}_a \phi^i \mathcal{D}_b \phi^i - \frac{1}{4} \sum_{ij} [\phi^i, \phi^j]^2 \right] \end{aligned} \tag{12}$$

$$G^{ab}\mathcal{D}_a\dot{\mathcal{A}}_b - i[\phi^i, \dot{\phi}^i] + \{\Psi^T, \Psi\} = 0 \qquad (13)$$

In the above we have defined the field strength as

$$\mathcal{F}_{ab} \equiv \partial_a\mathcal{A}_b - \partial_b\mathcal{A}_a - i[\mathcal{A}_a, \mathcal{A}_b] \qquad (14)$$

and covariant derivatives as

$$\mathcal{D}_a = \partial_a - i[\mathcal{A}_a, .] \qquad (15)$$

The "open string metric" G^{ab} is defined as

$$G^{ab} = \sum_c \theta^{ac}\theta^{bc} \qquad (16)$$

In the following we will restrict ourselves to the case $k = 2$ (that is, two coincident branes) and also $p = 1$ ($D2$ branes, see (5)) mostly. The resulting gauge theory will be a $U(2)$ noncommutative gauge theory in $2 + 1$ dimensions. We note that in this case

$$G^{ab} = \mathrm{diag}[\theta^2, \theta^2] \qquad (17)$$

The Brane-Antibrane System

We now consider the following static solution [32] to the equations of motion of $U(2)$ non-commutative gauge theory in $2 + 1$ dimensions:

$$\mathcal{A}_1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{2}{\theta}x^2 \end{pmatrix}, \quad \mathcal{F}_{12} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{\theta} \end{pmatrix},$$
$$\phi^j = 0 = \mathcal{A}_2 \qquad (18)$$

The fields are represented here as 2×2 matrices each entry of which is an operator in the Hilbert space $\mathcal{H}$.

The energy of this solution in the NCYM theory can be seen to be[4]

$$\mathcal{E} = \epsilon_0\, 2\,\mathrm{tr}\mathbf{1}, \quad \epsilon_0 \equiv \frac{\sqrt{2\pi}\theta^2}{2g_s} \qquad (19)$$

---

[4]We will use the notation "Tr" for trace over $U(\infty) \otimes U(2)$ indices whereas "tr" will denote a trace over only $U(\infty)$.

which is infinite. This is to be expected since we are considering noncompact 2-branes. A more useful notion is the tension of the brane-antibrane system, defined by

$$M = \int d^2x \mathcal{T}_2 = \mathcal{T}_2 \, 2\pi\theta \, \mathrm{tr}\mathbf{1} \tag{20}$$

where M is the rest mass, related to the light-front energy $\mathcal{E} = P^-$ by the formula [57, 63]

$$M^2 = 2\mathcal{E}P^+ \tag{21}$$

In order to calculate the tension, we need to regularize the expressions appearing in (19) and (20). Such a regularization is provided, e.g., by (8) in which the $U_\infty(2)$ representations are regarded as $2N \times 2N$ matrices with N sufficiently large. The light-front energy and rest mass then become

$$\mathcal{E} = \epsilon_0 \, 2N \tag{22}$$

$$M = \mathcal{T}_2 \, 2\pi\theta N \tag{23}$$

In our notation $\mathrm{tr}\mathbf{1} = N, \mathrm{Tr}\mathbf{1} = 2N$. Using the fact that for $2N \times 2N$ matrices, the light-cone momentum is

$$P^+ = \frac{2N}{R_{10}}, \quad R_{10} = \frac{g_s}{\sqrt{2\pi}}, \tag{24}$$

we get, from (21)

$$\mathcal{T}_2 = 2\mathcal{T}_{D2}, \quad \mathcal{T}_{D2} = \frac{1}{\sqrt{2\pi}g_s} = \frac{1}{(2\pi)^2 g_s l_s^3} \tag{25}$$

which is exactly the result expected from string theory [3]. In the final expression for the D2-brane tension $\mathcal{T}_{D2}$, we have reinstated l_s.

The solution (18) describes a coincident $D2 - \overline{D2}$ pair. It is easy to check, using (10), that the matrix model configuration equivalent to (18) is

$$X^1 = x^1 \otimes \mathbf{1}, X^2 = x^2 \otimes \sigma_3 \tag{26}$$

This is well-known [55, 56] to describe a coincident $D2 - \overline{D2}$ pair.

3   Unitary Gauge Fixing and the Tachyon Fields

In the discussion of tachyon condensation in the following three sections, we will work in the "Higgs branch" $\phi^j = 0$. We will relax this condition when we consider excitations around the tachyonic vacuum.

In this section, we discuss the procedure of gauge fixing in order to identify the physical degrees of freedom. We begin with a suggestive *parameterization* of the $U(2)$ variables which is useful:

$$\mathcal{A}_a(x) = \begin{pmatrix} A_a(x) & T_a(x) \\ T_a^\dagger(x) & \tilde{A}_a(x) \end{pmatrix} \tag{27}$$

The field strength is then given by:

$$\mathcal{F}_{ab} = \begin{pmatrix} F_{ab} - i(T_a T_b^\dagger - T_b T_a^\dagger) & D_a T_b - D_b T_a \\ (D_a T_b)^\dagger - (D_b T_a)^\dagger & \tilde{F}_{ab} - i(T_a^\dagger T_b - T_b^\dagger T_a) \end{pmatrix} \tag{28}$$

where $F_{12} = \partial_1 A_2 - \partial_2 A_1 - i[A_1, A_2]$, $\tilde{F}_{12} = \partial_1 \tilde{A}_2 - \partial_2 \tilde{A}_1 - i[\tilde{A}_1, \tilde{A}_2]$ and $D_a T_b = \partial_a T_b - iA_a T_b + iT_a \tilde{A}_b$.

We now discuss the unitary gauge that fixes the $U_\infty(2)$ gauge symmetry to $U_\infty(1) \times U_\infty(1)$. The unitary gauge, applied to the $U(2)$ field strength, says that the gauge field strength $\mathcal{F}_{12}$ is diagonal in the $U(2)$ space. This, in our parameterization, becomes

$$D_a T_b = D_b T_a \tag{29}$$

and its hermitian conjugate. It can be shown that every field configuration is gauge equivalent to one that satisfies the gauge condition (29) [5], and further the residual $U_\infty(1) \times U_\infty(1)$ gauge transformations are given by:

$$A_a \to U^\dagger A_a U + iU^\dagger \partial_a U \tag{30}$$

$$\tilde{A}_a \to \tilde{U}^\dagger A_a \tilde{U} + i\tilde{U}^\dagger \partial_a \tilde{U} \tag{31}$$

$$T_a \to U^\dagger T_a \tilde{U} \tag{32}$$

where $U, \tilde{U}$ are unitary operators on the noncommutative space (U here not to be confused with the U of (8)). These transformations enable us to interpret T_a as a matter field. While there appear to be two complex matter fields, the gauge fixing condition (29) relates them (we will detail the solution of the gauge fixing condition later on). The static part of the Hamiltonian (12), in the unitary gauge, is given by

$$\begin{aligned} H_{\text{st}} &= \epsilon_0 \, \text{tr} \, (h_1^2 + h_2^2) \\ h_1 &= \theta[F_{12} - i(T_1 T_2^\dagger - T_2 T_1^\dagger)] + \mathbf{1} \\ h_2 &= \theta[\tilde{F}_{12} - i(T_1^\dagger T_2 - T_2^\dagger T_1)] + \mathbf{1} \end{aligned} \tag{33}$$

---

[5] One can see this easily for large $2N \times 2N$ matrices.

The symbol ϵ_0 has been defined in (19). We will define the "potential" to be

$$V = H_{\text{st}} - \mathcal{E} = \epsilon_0 \left[\text{tr} \left(h_1^2 + h_2^2 \right) - 2 \, \text{tr} \, \mathbf{1} \right] \tag{34}$$

Eqn. (33) makes it obvious that the global minimum of the potential corresponds to

$$h_1 = h_2 = 0 \tag{35}$$

since the operators h_1, h_2 are self-adjoint. Thus, at the minimum of the potential, defined by (35), the total energy (33) of the static configuration, vanishes. We will shortly argue that this minimum corresponds to the phenomenon of tachyon condensation to the vacuum. Therefore, the potential at the minimum exactly cancels the initial energy (19) of the brane-antibrane system, leaving a state of zero energy. This is exactly as required by the Sen conjecture [2, 3]. We note that the existence of a simple supersymmetric hamiltonian (1),(12) makes it obvious that the ground state energy of the system should vanish. We also note that (35) is equivalent to, in terms of the original Matrix model variables,

$$[X^1, X^2] = 0 \tag{36}$$

In the following we will see that such ground states do correspond to the phenomenon of tachyon condensation to the "vacuum".

It is useful to write the potential in the Moyal star-product notation, as

$$\begin{aligned} H_{\text{st}} &= \int d^2x \, h, \\ h &= 2\pi\theta \, \epsilon_0 \, (h_1 \star h_1 + h_2 \star h_2) \end{aligned} \tag{37}$$

where h_1, h_2 are given by (33). Clearly at this stage the static hamiltonian is translationally invariant.

In order to obtain the Hamiltonian in terms of independent degrees of freedom we must further fix the $U(1) \times U(1)$ gauge symmetry, and solve the Gauss law condition (13). We reiterate that the above formulation is as yet completely general and without reference to any specific classical configuration of the gauge theory.

Explicit vacuum solutions

9

To show that there exist states with zero energy, we explicitly exhibit a class of such states:

$$A_a = \tilde{A}_a = 0$$
$$T_1 = ax^2 + bx^1, \; T_2 = cx^2 + dx^1, \; ad - bc = 1 \tag{38}$$

It is straightforward to check that the above configuration satisfies (35). The case $(a, b, c, d) = (0, -1, 1, 0)$ is discussed in [32].

In Section 5, we will encounter other examples of vacuum solutions. In fact the most general solution for $A_a, \tilde{A}_a$, and T_a that corresponds to the ground state can be explicitly inferred by using (36) and (10) in the following way: (a) choose a basis of the Hilbert space $\mathcal{H}$, (b) take X^1, X^2 to be arbitrary diagonal matrices in this basis, (c) express x^1, x^2 in this basis, (d) find $A_a, \tilde{A}_a, T_a$ using (10).

4 The Exact Potential

We now discuss how to fix the $U(1) \times U(1)$ gauge symmetry. We can, for instance, choose the background gauge (corresponding to the background (18)):

$$\partial_1 A_1 + \partial_2 A_2 = 0$$
$$\partial_1 \tilde{A}_1 - i\frac{2}{\theta}[x^2, \tilde{A}_1] + \partial_2 \tilde{A}_2 = 0 \tag{39}$$

where by $A_a, \tilde{A}_a$ we imply the fluctuations. It is easy to see that such a gauge choice is translationally invariant, that is, invariant under $x^a \to x^a + b^a \mathbf{1}$. If we are able to solve (39) for the independent variables, the resulting hamiltonian will be translationally invariant.

It turns out to be difficult, however, to solve (29) and (39) for the independent variables. Let us, instead, fix the residual $U(1) \times U(1)$ symmetry by imposing the axial gauge

$$A_1 = 0, \; \tilde{A}_1 = \frac{2x^2}{\theta} \tag{40}$$

Note that any configuration of $A_a, \tilde{A}_a, T_a$ can be brought to a configuration satisfying (40) by applying the gauge transformations (32). We will explicitly show this for four examples:

10

(1) The classical configuration corresponding to two $D2$ branes, namely

$$A_1 = A_2 = 0, \ \tilde{A}_1 = \tilde{A}_2 = 0, \ T_1 = T_2 = 0 \tag{41}$$

can be put in the form

$$A_1 = A_2 = 0, \ \tilde{A}_1 = \frac{2x_2}{\theta}, \tilde{A}_2 = -\frac{2x_1}{\theta}, \ T_1 = T_2 = 0 \tag{42}$$

by choosing

$$U = 1, \ \tilde{U} = \exp[i\pi \frac{(x^1)^2 + (x^2)^2}{2\theta}] \tag{43}$$

in (32).

(2) By using the same transformation (43) the vacuum solution (38) can be put in the form

$$
\begin{aligned}
A_1 &= A_2 = 0, \ \tilde{A}_1 = \frac{2x_2}{\theta}, \tilde{A}_2 = -\frac{2x_1}{\theta}, \\
T_1 &= (ax^2 + bx^1)\tilde{U}, \ T_2 = (cx^2 + dx^1)\tilde{U}
\end{aligned}
\tag{44}
$$

where $\tilde{U}$ is as in (43).

(3) The n-vortex solutions (63) that we will encounter later can also be recast, using (43), as

$$
\begin{aligned}
A_1 &= A_2 = 0, \ \tilde{A}_1 = \frac{2x_2}{\theta}, \tilde{A}_2 = -\frac{2x_1}{\theta}, \\
T_1 &= (\sum_{j=0}^{n-1} t_j P_j)\tilde{U}, \ T_2 = (\sum_{j=0}^{n-1} \omega_j P_j)\tilde{U}
\end{aligned}
\tag{45}
$$

where the t_j, ω_j's are defined by (67),(69),(70). Clearly the end-point (75), the $2N$-vortex vacuum, can also be put in the axial gauge (40).

(4) The fourth example is the brane-antibrane configuration (18) which already satisfies (40).

The point of the above exercise was to emphasize that (40) is only a choice of gauge, and not a choice of background. Indeed the explicit appearance of examples (2), (3) and (4) imply that we can describe the entire process of tachyon condensation in this gauge. If we are able to solve (40) exactly, which we will, the resulting hamiltonian (namely, (51)) will, therefore, correspond to an *exact* hamiltonian for tachyon condensation.

11

Let us, therefore, proceed to solve (40), together with (29). Using the Moyal product notation, we get

$$D_1 T_2 - D_2 T_1 = 0 = \partial_1 T_2 + \frac{2i}{\theta} T_2 \star x^2 - D_2 T_1 \qquad (46)$$

where

$$D_2 T_1 = \partial_2 T_1 - i A_2 \star T_1 + i T_1 \star \tilde{A}_2. \qquad (47)$$

Using the fact that

$$\partial_1 T_2 + \frac{2i}{\theta} T_2 \star x^2 = \frac{2i}{\theta} x^2 T_2, \qquad (48)$$

we can solve (46) explicitly:

$$T_2 = \frac{\theta}{2 i x^2} (\partial_2 T_1 - i A_2 * T_1 + i T_1 * \tilde{A}_2) \qquad (49)$$

Note that the independent physical variables are $T_1, A_2, \tilde{A}_2$, in other words, one complex matter field and two real gauge fields. This is of course exactly what one expects from the spectrum of open string theory on the brane-antibrane system. In the following we will denote

$$T_1 = T, A_2 = A, \tilde{A}_2 = \tilde{A} \qquad (50)$$

In order to make the division by x^2 in (49) well-defined, one can use the prescription $1/x^2 \rightarrow 1/(x^2 + i\epsilon)$. This agrees with the explicit solutions for T_2 presented in the next section.

Substituting this solution into (33), we get the exact static Hamiltonian in terms of the independent variables $T, A, \tilde{A}$:

$$
\begin{aligned}
H_{st} = \ & \epsilon_0 \int \frac{dx^1 dx^2}{2\pi\theta} \Bigg\{ \left(\theta \partial_1 A + \frac{\theta^2}{2} \Big[T * (\frac{1}{x^2} D_2 \bar{T}) + (\frac{1}{x^2} D_2 T) * \bar{T} \Big] + 1 \right)^2 \\
& + \left(\theta \partial_1 \tilde{A} - \frac{\theta^2}{2} \Big[\bar{T} * (\frac{1}{x^2} D_2 T) + (\frac{1}{x^2} D_2 \bar{T}) * T \Big] - 1 \right)^2 \Bigg\} \quad (51)
\end{aligned}
$$

where the bar denotes complex conjugation. In the above equation,

$$D_2 T = \partial_2 T - i A \star T + i T \star \tilde{A} \qquad (52)$$

and the squares are evaluated using star products. ϵ_0 is defined in (33).

The potential (34) is given by

$$
V = \epsilon_0 \int \frac{dx^1 dx^2}{2\pi\theta} \left\{ \left(\theta\partial_1 A + \frac{\theta^2}{2}\left[T * (\frac{1}{x^2}D_2\bar{T}) + (\frac{1}{x^2}D_2 T) * \bar{T} \right] + 1 \right)^2 - 1 \right.
$$
$$
\left. + \left(\theta\partial_1\tilde{A} - \frac{\theta^2}{2}\left[\bar{T} * (\frac{1}{x^2}D_2 T) + (\frac{1}{x^2}D_2\bar{T}) * T \right] - 1 \right)^2 - 1 \right\} \quad (53)
$$

It is interesting to note that the gauge fields $A, \tilde{A}$ appear quadratically in (51),(53). Thus, it is possible to eliminate them exactly and get an effective potential in terms of T alone. We will not attempt to do it in this paper.

Gauss' law constraint

It is easy to see that time-independent bosonic configurations automatically satisfy (13). Therefore the static hamiltonian and the potential mentioned above are consistent with the Gauss' law constraint. In Section 6 where we will consider time-dependent ϕ^3, we will restrict to configurations satisfying $[\phi^3, \dot{\phi}^3] = 0$, hence satisfying Gauss' law once again.

5   Vortex production and brane annihilation

In order to gain more insight into the nature of the potential (53) and various static solutions, it is useful to begin with simple classes of configurations of $T, A, \tilde{A}$.

Case 1 (simple vortex):

Let us consider a configuration space defined by

$$
T = tP_0, \; A = aP_0, \; \tilde{A} = \tilde{a}P_0, \quad t \in \mathbf{C}, a, \tilde{a} \in \mathbf{R} \quad (54)
$$

where $P_0 = |0\rangle\langle 0|$, described in the Moyal form as

$$
P_0 = 2\exp\left[-\frac{r^2}{\theta} \right], \quad r^2 = (x^1)^2 + (x^2)^2 \quad (55)
$$

If we further specify $a = \tilde{a}$, the dependent variable T_2 in (49) becomes also proportional to P_0, with $T_2 = i\,tP_0$, leading to a consistent truncation of all the fields to the

one-dimensional subspace P_0 of operators. The resulting configuration space clearly corresponds to a single localized vortex at the origin of the (x^1, x^2) plane. We will shortly confirm this by explicitly calculating the vortex charge.

Let us now find out the minimum of the static hamiltonian H_{st} (51) or the potential (53) in the configuration (54). It turns out that calculations are easier in this example in the operator formulation in which (46) reads as

$$\{x^2, T_2\} = -[x^1, T_1] - A_2 T_1 + T_1 \tilde{A}_2 \tag{56}$$

The result is:

$$\frac{H_{\text{st}}(t, a)}{\epsilon_0} = 2[N - 4\theta|t|^2 + 4\theta^2|t|^4 + \theta a^2] \tag{57}$$

$$\frac{V(t, a)}{\epsilon_0} = 2[-4\theta|t|^2 + 4\theta^2|t|^4 + \theta a^2] \tag{58}$$

Here we have regularized $\text{tr}\mathbf{1} = N$ by using an N-dimensional Hilbert space, described by, $e.g$, (8). Note the negative sign of the $|t|^2$, reflecting a tachyonic mode. Indeed the tachyonic part of the hamiltonian (1), including the kinetic term, turns out to be

$$H = 4\epsilon_0 \left(|\dot{t}|^2 - 2\theta|t|^2 + 2\theta^2|t|^4\right) \tag{59}$$

which was obtained in [55] by explicit Matrix model calculations. Equation (59) also appears in [32] (Eq. (35)) whose T is related to our parameter t by $T = 2t\sqrt{\theta}$. The mass $m_T^2 = -2\theta$ agrees with the string theory result [32] $\alpha' m_T^2 = -\frac{1}{\pi b}$, $b = B_{12}$.

The minimum of the potential is easily found by noting that

$$\frac{V(t, a)}{\epsilon_0} + 2 = 2[(1 - 2\theta|t|^2)^2 + \theta a^2] \tag{60}$$

The minimum value of the right hand side is zero, which occurs at

$$|t|^2 = \frac{1}{2\theta}, a = 0. \tag{61}$$

Thus

$$\begin{aligned}
\frac{V(t, a)_{\text{min}}}{\epsilon_0} &= -2 \\
\frac{H_{\text{st}}(t, a)_{\text{min}}}{\epsilon_0} &= 2N - 2
\end{aligned} \tag{62}$$

14

This clearly shows that the subspace (54) describes formation of a single vortex and the classical vortex configuration (61) cancels 2 units out of 2N units of total energy of the brane-antibrane configuration.

Case 2 (multiple vortices):

The above result can be easily generalized to an n-vortex configuration space, given by

$$T = \sum_{j=0}^{n-1} t_j P_j, \ A = \tilde{A} = \sum_{j=0}^{n-1} a_j P_j \tag{63}$$

where

$$P_j = |j\rangle\langle j| \tag{64}$$

Note that Eq. (63), in the Moyal notation, corresponds to the spherically symmetric ansatz

$$T(x^1, x^2) = \sum_{j=0}^{n-1} t_j 2(-1)^j L_j(2r^2/\theta)e^{-r^2/\theta} \tag{65}$$

Once again, the choice $A = \tilde{A}$ ensures that the dependent variable T_2 is of the same form as T, namely,

$$T_2 = \sum_{j=0}^{n-1} i\,\omega_j P_j \tag{66}$$

The gauge fixing conditions (46) are solved by

$$\omega_{n-1} = t_{n-1}$$
$$\omega_{n-2} = (t_{n-2} - 2t_{n-1})$$
$$\ldots$$
$$\omega_1 = \left(t_1 - 2(t_2 + \ldots + (-1)^{n-1}t_{n-1})\right)$$
$$\omega_0 = \left(t_0 - 2(t_1 + \ldots - (-1)^{n-1}t_{n-1})\right) \tag{67}$$

The static hamiltonian and the potential are straightforward to calculate. One gets

$$\frac{H_{\text{st}}}{\epsilon_0} = 2\left[\sum_{j=0}^{n-1}(1 - \theta t_j \bar{\omega}_j - \theta \bar{t}_j \omega_j)^2 + \sum_{j=0}^{n-2} \theta(a_j - a_{j+1})^2 + \theta a_{n-1}^2\right] + 2(N-n)$$
$$\frac{V}{\epsilon_0} = \frac{H_{\text{st}}}{\epsilon_0} - 2N \tag{68}$$

15

It is straightforward to write out these expressions for small n, and discover the tachyonic directions; we will not write the explicit expressions here.

Since the terms in the square brackets in the hamiltonian is a sum of squares, the minimum possible value is zero. We explicitly find the minimum, for real t_j, to be

$$\begin{aligned} a_j &= 0 \\ t_j &= \frac{1}{\sqrt{\theta}} r_j, \; j = 0, \ldots, n-1 \end{aligned} \tag{69}$$

where the r_j are defined by

$$2r_{n-1}^2 - 1 = 0$$
$$2r_{n-2}^2 - 4r_{n-2}r_{n-1} - 1 = 0$$
$$\ldots$$
$$2r_0^2 - 4r_0(r_1 - r_2 + \ldots + (-1)^n r_{n-1}) - 1 = 0 \tag{70}$$

It is easy to see that real roots always exist (the discriminants are always non-negative). For example,

$$r_{n-1} = \frac{1}{\sqrt{2}}, \; r_{n-2} = \frac{1}{\sqrt{2}} + 1, \text{ etc.} \tag{71}$$

The potential and the total energy, evaluated at the minimum, is given by

$$\begin{aligned} \frac{V(\vec{t}, \vec{a})_{\min}}{\epsilon_0} &= -2n \\ \frac{H_{\text{st}}(\vec{t}, \vec{a})_{\min}}{\epsilon_0} &= 2N - 2n \end{aligned} \tag{72}$$

Thus the n-vortex solution cancels exactly $2n$ out of the total of $2N$ units of energy of the brane-antibrane system.

Topological charge of the vortex:

In [18] the topological charge of vortex configurations for a complex tachyon (assumed to be governed by NC abelian Higgs model) was found by essentially looking for the "surface term" in the hamiltonian in the Bogomolnyi framework. Repeating a similar analysis here, we find that the topological charge of these vortex solutions are given by

$$\mathcal{I} = \text{Tr}\mathcal{F}_{12} \tag{73}$$

16

We find that the vortex charge for the solution (63),(69) is exactly

$$\mathcal{I} = 2n. \tag{74}$$

The factor 2 appears because we have normalized the topological charge to reproduce the $D0$ charge. Recall that a vortex of the complex tachyon on a $Dp\text{-}\overline{Dp}$ system gives rise to BPS $D(p-2)$ branes [1] and the RR charge of the latter is supposed to arise as the topological charge of the vortex. The latter fact has been explicitly shown in the complex tachyon model studied in [18].

The vacuum:

From the above discussion it is clear that the tachyonic vacuum is given by $n = N$ in (63). To be explicit, it is given by

$$T = \sum_{j=0}^{N-1} t_j P_j, \ A = \tilde{A} = 0 \tag{75}$$

where t_j's are defined by (69),(70).

Picture of gradual annihilation:

The preceding discussion provides rather compelling evidence, within the framework of $U(2)$ NC gauge theory, that the tachyon condensate consisting of $2N$ vortices, exactly cancels the energy of the brane-antibrane system. This, therefore, proves Sen's conjecture. Furthermore, we also see a concrete mechanism, within the gauge theory, of successively lowering the energy of the system by vortex production. The picture here is that the brane-antibrane pair is gradually annihilating, creating more and more vortices (which, through their topological charge, are identified as $D0$-branes). To see this picture concretely, let us note that for the $2n$-vortex solution (63),(69)

$$\frac{1}{i\theta}[X^1, X^2] = \begin{pmatrix} \mathbf{0}_{2n} & & \\ & \mathbf{1}_{N-n} & \\ & & -\mathbf{1}_{N-n} \end{pmatrix} \tag{76}$$

Since $[X^1, X^2]$ represents $D2$-brane charge density, and since commuting X^1, X^2 represents $D0$ branes, (76) represents "partial annihilation" of the brane-antibrane system to

17

D0-branes. The lower two blocks represent the remaining brane-antibrane system and the lower $2n \times 2n$ block represents the D0-branes released by partial annihilation. Note that the brane-antibrane configuration (18) and the tachyonic vacuum (cf. (36)) correspond respectively to

$$\frac{1}{i\theta}[X^1, X^2]_{D2\,\overline{D2}} = \begin{pmatrix} \mathbf{1}_{N-n} & \\ & -\mathbf{1}_{N-n} \end{pmatrix}, \quad \frac{1}{i\theta}[X^1, X^2]_{\text{vacuum}} = \begin{pmatrix} \mathbf{0}_{2N} \end{pmatrix} \qquad (77)$$

6 Closed string excitations around the tachyonic vacuum

In the previous sections we have described several degenerate solutions of the tachyonic vacuum (e.g (38), (75). To analyze the spectrum of fluctuations around any of these, one would normally write the action or the hamiltonian of the noncommutative gauge theory around this vacuum upto quadratic order in the variables $T, A, \tilde{A}, \phi^j$. We will find it simpler, however, to go back to the Matrix model variables $X^a, a = 1, 2$ and $X^i, i = 3, 4, \ldots, 9$ and analyze the quadratic fluctuations in terms of them. The tachyonic vacuum, as we have seen (36), is characterised by

$$[X^1, X^2] = 0, \; X^i = 0 \qquad (78)$$

Clearly, there is a diagonal basis in which this will look like

$$X^1 = \Lambda^1 \equiv \text{diag}[\lambda^1_{(1)}, \lambda^1_{(2)}, \ldots], \; X^2 = \Lambda^2 \equiv \text{diag}[\lambda^2_{(1)}, \lambda^2_{(2)}, \ldots], \; X^i = 0 \qquad (79)$$

The eigenvalues $\vec{X}_{nn} = (\lambda^1_{(n)}, \lambda^2_{(n)}, 0, \ldots, 0), \; n = 1, \ldots, 2N$ represent coordinates of the $2N$ D0 branes in the vacuum.

We will look for a closed string in the x^9 direction, perpendicular to the (annihilated) brane-antibrane system. In order to describe closed strings with a finite energy, we will compactify the x^3 direction on a circle of finite radius R_9. Matrix theory on a circle is described [62] by assigning appropriate periodicity properties to the matrices X^M. The hamiltonian (1) becomes (after dropping the fermions)

$$H = \frac{1}{g_s l_s} \int_0^{2\pi} \frac{d\sigma}{2\pi} \, \text{Tr} \left(\frac{R_9^2}{2}(\dot{A})^2 + \frac{1}{2} \sum_{i=1}^{8} \left[(\dot{Y}^i)^2 + \frac{1}{(2\pi l_s^2)^2} \left(R_9^2 (DY^i)^2 - \frac{1}{2} \sum_j [Y^i, Y^j]^2 \right) \right] \right) \qquad (80)$$

18

which can be obtained from the BFSS hamiltonian (1) by (i) making the replacements [62]

$$X^9 \to R_9\, iD \equiv R_9(i\partial_\sigma + A(\sigma)), \quad \dot{X}^3 \to R_9\,\dot{A}(\sigma), \quad X^i \to Y^i(\sigma), i = 1, 2, \ldots 8 \quad (81)$$

and (ii) reinstating l_s. It is useful to recast this in another form. Let us (a) substitute in (80) the velocities $\dot{A}, \dot{Y}^i$ by their conjugate momenta, defined by

$$E = \frac{R_9^2}{g_s l_s}\dot{A}, \Pi_i = \frac{1}{g_s l_s}\dot{Y}^i \quad (82)$$

(b) replace g_s, l_s by the M-theory variables R_{10}, l_P (we denote the M-theory coordinates as $x^0, \ldots, x^9, x^{10}$) using the relation

$$R_{10} = g_s l_s, l_P = g_s^{1/3} l_s; \quad g_s = (R_{10}/l_P)^{3/2}, l_s = l_P^{3/2} R_{10}^{-1/2} \quad (83)$$

and (c) rescale

$$Y^i \to (\frac{2\pi l_P^3}{R_9})^{1/2}, \Pi_i \to (\frac{2\pi l_P^3}{R_9})^{-1/2}\Pi_i \quad (84)$$

We get

$$H = \int_0^{2\pi}\frac{d\sigma}{2\pi} \left(\text{Tr}\Big[\frac{R_9 R_{10}}{4\pi l_P^3}\sum_{i=1}^8 (\Pi_i^2 + (DY^i)^2) + (\frac{R_{10}}{2R_9^2})^3(E^2 - \frac{1}{2}\sum_{i,j}[Y^i, Y^j]^2)\Big]\right) \quad (85)$$

The hamiltonian (85) can be regarded as a description of 11-dimensional M-theory on a torus of radii (R_9, R_{10}) (in which R_{10} denotes a light-like compactification [63, 64, 65] of M-theory, with states characterized by the light-like momentum $P^+ = \frac{2N}{R_{10}}$).

The Gauss law constraint (2), in the variables of (80), reads

$$i\partial_\sigma \dot{A} + [A, \dot{A}] + \frac{1}{R_9}[Y^i, \dot{Y}^i] = 0 \quad (86)$$

The vacuum of (80) should satisfy

$$\dot{A} = 0, i\partial_\sigma Y^i + [A, Y^i] = 0, \dot{Y}^i = 0, [Y^i, Y^j] = 0 \quad (87)$$

The tachyonic vacuum (79), which in the variables of (80), reads

$$A(\sigma) = 0, Y^1(\sigma) = \Lambda^1, Y^2(\sigma) = \Lambda^2, Y^i(\sigma) = 0, i = 3, \ldots, 8, \quad (88)$$

clearly satisfies (87).

19

We will consider now fluctuations of (80) or (85) around the tachyonic vacuum (88). We will consider two special regions of the parameter space (R_9, R_{10}):

Region A: $R_{10} \ll l_P, R_9 \sim l_P$

Region B: $R_9 \ll l_P, R_{10} \sim l_P$,

Using (83) it is clear that the original weakly coupled NC gauge theory refers to Region A. We will however find it more convenient to consider region B first.

Region B:

The analysis of (85) in this region is exactly the same as in [60]. We will, therefore, mention only the salient points. In this region it is more appropriate to obtain a type IIA theory by compactifying M-theory on x^9 (rather than on x^{10}). The string length $\tilde{l}_s$ and string coupling $\tilde{g}_s$ are given by

$$R_9 = \tilde{g}_s \tilde{l}_s, l_P = \tilde{g}_s^{1/3} l \tilde{l}_s; \quad \tilde{g}_s = (R_9/l_P)^{3/2}, \tilde{l}_s = l_P^{3/2} R_9^{-1/2} \tag{89}$$

Clearly $\tilde{g}_s \ll 1$. As shown by [60], in this limit, the off-diagonal fluctuations of Y^i are heavy, so that $[Y^i, Y^j] = 0$. The fluctuations of E are also heavy. The lightest sector is described by a $1 + 1$ dimensional conformal field theory on $S^{2N}(R^8)$. The states of this theory are a collection of weakly coupled closed strings (coupling $\tilde{g}_s$) of various lengths n_i with $\sum_i n_i = 2N$. The mass scale of these strings is

$$\tilde{m}_s = \frac{1}{\tilde{l}_s} = \frac{R_9^{1/2}}{l_P^{3/2}} \tag{90}$$

As mentioned in [60], these closed string states are BPS states. Therefore these states exist for all values of the coupling and we can continue the mass formula beyond Region B, in particular to Region A. In this region, the weakly coupled IIA theory is obtained by original compactification on x^{10}, described by (83). (90) becomes

$$\tilde{m}_s = \frac{R_9^{1/2}}{g_s^{1/2} l_s^{3/2}} \tag{91}$$

which is clearly a massive state for small g_s. These are therefore solitonic states in the theory with small g_s.

It is straightforward to see that the tachyonic vacuum (88) corresponds to the ground state of the the superconformal field theory on $S^{2N}(R^8)$.

20

This, therefore, establishes that there are solitonic closed string states around the tachyonic vacuum.

Region A:

As we remarked already, this region corresponds to the original coupling g_s being weak. In this region the closed string states constructed above using the transverse coordinates Y^i are heavy. We will find light states here which correspond to electric flux lines along x^9. These are similar to the ones found for unstable Dp branes.

We begin, therefore, by considering fluctuations of $A(\sigma)$ in (80) around the tachyonic vacuum (88). For a generic choice of the eigenvalues of Λ^1, Λ^2 the $U(2N)$ symmetry will be completely broken to $U(1)$ factors. In this case, the $[A, Y^1]^2 + [A, Y^2]^2$ term will give mass to the off-diagonal fluctuations of A. Since we are looking for the lightest states, we will therefore restrict to A of the diagonal form

$$A = \Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_{2N}] \tag{92}$$

For such a gauge field, the Gauss law reads (in the absence of fluctuations of the Y^i's)

$$\partial_\sigma \dot{\Lambda} = 0 \tag{93}$$

With the above ingredients, the hamiltonian (80) becomes

$$H = \frac{R_9^2}{2g_s l_s} \sum_{i=1}^{2N} (\dot{\lambda}_i)^2 = \frac{g_s l_s}{2R_9^2} \sum_{i=1}^{2N} e_i^2 \tag{94}$$

where the canonical momenta (electric field eigenvalues) e_i are given by

$$e_i = \frac{R_9^2}{g_s l_s} \dot{\lambda}_i \tag{95}$$

We will show in a simple example that the above spectrum of electric fluxes describes closed string states in the dual type IIB description. Recall that the BFSS hamiltonian (80) describes a system of $2N$ D-strings in a dual type IIB theory described by a radius R_9' and coupling constant g_s'

$$R_9' = \frac{l_s^2}{R_9}, \quad g_s' = g_s \frac{l_s}{R_9} \tag{96}$$

Consider a bound state of n fundamental strings and a single D-string (say the first one). This is described by an electric flux e_1 on the world-volume of the first D-string (and no flux on the others), satifying

$$e_1 = n, e_i = 0, i = 2, \ldots 2N \tag{97}$$

The energy of such a configuration, according to (94), is

$$\mathcal{E} = \frac{g_s l_s}{2R_9^2} n^2 = g_s' \frac{R_9'}{2l_s^2} n^2 \tag{98}$$

From the above (light-front) energy we can get the rest mass M by using (21) with $P^+ = 1/R$ (corresponding to a single D string carrying the flux). The result is

$$M = n \frac{2\pi R_9'}{2\pi l_s^2} \tag{99}$$

which is exactly the mass of a IIB string of winding number n.

This proves that the state we have constructed above is a closed string state. In the type IIB description, these are winding states, whereas in the type IIA description these will correspond to Kaluza-Klein states. For an (n, k) bound state one needs to take into account the permutation group $S(k)$ which played a role in our discussion of the solitonic strings. Our result above can be generalized to this case by arguments similar to the ones used in [60] to describe bound states of $D0$ branes and Matrix strings.

We note that if $\Lambda^{1,2}$ are taken to be diagonal in the simple harmonic oscillator basis, the flux configuration (97) corresponds to a localized functions (given by Laguerre polynomials) in the (x^1, x^2) plane. This corresponds to a closed flux tube (along x^9) having a finite width in the (x^1, x^2) directions.

7   Discussion

We have shown in this paper that tachyon condensation can be described in the context of brane-antibrane pairs in a well-defined Hamiltonian framework (1) with a supersymmetric ground state. The existence of such a Hamiltonian framework guarantees that Sen's conjecture must be true, as we have found out explicitly. We have further shown that there are two types of closed string excitations around the tachyonic vacuum. The first

type is solitonic (mass $\propto g_s^{-1/2}$) which are essentially the Matrix strings of [60]. The second type are flux lines whose masses are small.

We conclude with a couple of remarks:

(1) It has been noted [2, 39, 34, 35, 40] that the phenomenon of tachyon condensation incorporates the Higgs mechanism. The "symmetric vacuum" (top of the potential) of the Higgs model corresponds to the perturbative vacuum of open string theory on the brane; at the stable vacuum the gauge fields, which are open string fluctuations, are eaten up by the Higgs mechanism. It has been suggested recently [66] that in fact the *entire tower* of open strings is "Higgsed", giving rise to closed string degrees of freedom. This viewpoint is rather interesting, and a thorough understanding of such a claim would appear to require open string field theory. Since $U_\infty(k)$ NC gauge theory on branes, equivalently the Matrix model formulation, incorporates many features of open string field theory, it seems to provide an ideal set-up for addressing the stringy Higgs mechanism. It will be very interesting from this point of view to analyze the complete set of fluctuations of the NC gauge theory described in this paper both at the top of the potential as well as at the tachyonic vacuum.

(2) We have written down an exact tachyon potential (53) in this paper, including the gauge fields. It would be interesting to compare our result with the various proposals for action for brane-antibrane pairs, e.g. in [40, 37]. It is also interesting to see how our action connects to the ones describing the case of the real tachyon on unstable Dp-branes.

(3) The discussion in our paper has been mostly on a single brane-antibrane pair. The generalization to k brane-antibrane pairs is an important problem. As explained in Section 2, this involves $U(2k)$ noncommutative gauge theory. The brane-antibrane system is an unstable, nonsupersymmetric state of this gauge theory. It is important to study the large k limit of these theories and study the tachyon potential as well as explore possible connection with nonsupersymmetric black holes.

References

[1] A. Sen, "Descent relations among bosonic D-branes," Int. J. Mod. Phys. **A14**, 4061 (1999) [hep-th/9902105].

[2] A. Sen, "Stable non-BPS bound states of BPS D-branes," JHEP **9808**, 010 (1998) [hep-th/9805019]; "Tachyon condensation on the brane antibrane system," JHEP **9808**, 012 (1998) [hep-th/9805170]; "BPS D-branes on non-supersymmetric cycles," JHEP **12**, 021 (1998) hep-th/9812031.

[3] A. Sen, "Non-BPS states and branes in string theory," hep-th/9904207.

[4] E. Witten, "D-branes and K-theory," JHEP **12**, 019 (1998) hep-th/9810188; "Overview of K-theory applied to strings," hep-th/0007175.

[5] P. Horava, "Type IIA D-branes, K-theory, and matrix theory," Adv. Theor. Math. Phys. **2**, 1373 (1999) hep-th/9812135.

[6] A. Recknagel and V. Schomerus, "Boundary deformation theory and moduli spaces of D-branes," Nucl. Phys. **B545**, 233 (1999) hep-th/9811237.

[7] C.G. Callan, I.R. Klebanov, A.W. Ludwig and J.M. Maldacena, "Exact solution of a boundary conformal field theory," Nucl. Phys. **B422**, 417 (1994) hep-th/9402113.

[8] J. Polchinski and L. Thorlacius, "Free fermion representation of a boundary conformal field theory," Phys. Rev. **D50**, 622 (1994) hep-th/9404008.

[9] J. A. Harvey, P. Horava and P. Kraus, "D-sphalerons and the topology of string configuration space," JHEP **0003**, 021 (2000) [hep-th/0001143].

[10] J. A. Harvey and P. Kraus, "D-branes as unstable lumps in bosonic open string field theory," JHEP **0004**, 012 (2000) [hep-th/0002117].

[11] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, "Lumps and p-branes in open string field theory," Phys. Lett. **B482**, 249 (2000) [hep-th/0003031].

[12] N. Moeller, A. Sen and B. Zwiebach, "D-branes as tachyon lumps in string field theory," JHEP **0008**, 039 (2000) [hep-th/0005036].

[13] R. de Mello Koch and J. P. Rodrigues, "Lumps in level truncated open string field theory," hep-th/0008053.

[14] N. Moeller, "Codimension two lump solutions in string field theory and tachyonic theories," hep-th/0008101.

[15] K. Dasgupta, S. Mukhi and G. Rajesh, "Noncommutative tachyons," JHEP **0006**, 022 (2000) [hep-th/0005006].

[16] J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, "D-branes and strings as non-commutative solitons," JHEP **0007**, 042 (2000) [hep-th/0005031].

[17] E. Witten, "Noncommutative tachyons and string field theory," hep-th/0006071.

[18] D. P. Jatkar, G. Mandal and S. R. Wadia, "Nielsen-Olesen vortices in noncommutative Abelian Higgs model," hep-th/0007078.

[19] Y. Hikida, M. Nozaki and T. Takayanagi, "Tachyon condensation on fuzzy sphere and noncommutative solitons," hep-th/0008023.

[20] J. A. Harvey, P. Kraus and F. Larsen, "Tensionless branes and discrete gauge symmetry," hep-th/0008064.

[21] G. Mandal and S. Rey, "A note on D-branes of odd codimensions from noncommutative tachyons," hep-th/0008214.

[22] V. A. Kostelecky and S. Samuel, "On A Nonperturbative Vacuum For The Open Bosonic String," Nucl. Phys. **B336**, 263 (1990).

[23] A. Sen and B. Zwiebach, "Tachyon condensation in string field theory," JHEP **0003**, 002 (2000) [hep-th/9912249].

[24] N. Berkovits, A. Sen and B. Zwiebach, "Tachyon condensation in superstring field theory," hep-th/0002211.

[25] N. Moeller and W. Taylor, "Level truncation and the tachyon in open bosonic string field theory," Nucl. Phys. **B583**, 105 (2000) [hep-th/0002237].

[26] P. De Smet and J. Raeymaekers, "Level four approximation to the tachyon potential in superstring field theory," JHEP **0005**, 051 (2000) [hep-th/0003220]; "The tachyon potential in Witten's superstring field theory," JHEP **0008**, 020 (2000) [hep-th/0004112].

[27] L. Rastelli and B. Zwiebach, "Tachyon potentials, star products and universality," hep-th/0006240.

[28] V. A. Kostelecky and R. Potting, "Analytical construction of a nonperturbative vacuum for the open bosonic mstring," hep-th/0008252.

[29] R. Gopakumar, S. Minwalla and A. Strominger, "Symmetry restoration and tachyon condensation in open string theory," hep-th/0007226.

[30] A. Sen, "Some issues in non-commutative tachyon condensation," hep-th/0009038.

[31] R. Tatar, "A note on non-commutative field theory and stability of brane-antibrane systems," hep-th/0009213.

[32] P. Kraus, A. Rajaraman and S. Shenker, "Tachyon condensation in noncommutative gauge theory," hep-th/0010016.

[33] M. Li, "Note on noncommutative tachyon in matrix models," hep-th/0010058.

[34] A. Sen, "Supersymmetric world-volume action for non-BPS D-branes," JHEP **9910** (1999) 008 [hep-th/9909062].

[35] A. Sen, "Universality of the tachyon potential," JHEP **9912** (1999) 027 [hep-th/9911116].

[36] A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory," hep-th/0009103.

[37] D. Kutasov, M. Marino and G. Moore, "Some exact results on tachyon condensation in string field theory," hep-th/0009148.

[38] S. Dasgupta and T. Dasgupta, "Renormalization group analysis of tachyon condensation," hep-th/0010247.

[39] P. Yi, "Membranes from five-branes and fundamental strings from Dp branes," Nucl. Phys. **B550**, 214 (1999) [hep-th/9901159].

[40] O. Bergman, K. Hori and P. Yi, "Confinement on the brane," Nucl. Phys. **B580**, 289 (2000) [hep-th/0002223].

[41] G. Gibbons, K. Hori and P. Yi, "String fluid from unstable D-branes," hep-th/0009061.

[42] F. Larsen, "Fundamental Strings as Noncommutative Solitons," hep-th/0010181.

[43] A. Sen, "Fundamental strings in open string theory at the tachyonic vacuum," hep-th/0010240.

[44] A. Connes, M. R. Douglas and A. Schwarz, "Noncommutative geometry and matrix theory: Compactification on tori," JHEP **9802**, 003 (1998) [hep-th/9711162].

[45] N. Seiberg and E. Witten, "String theory and noncommutative geometry," JHEP **9909**, 032 (1999) [hep-th/9908142].

[46] R. Gopakumar, S. Minwalla and A. Strominger, "Noncommutative solitons," JHEP **0005**, 020 (2000) [hep-th/0003160].

[47] D. J. Gross and N. A. Nekrasov, "Monopoles and strings in noncommutative gauge theory," JHEP **0007** (2000) 034 [hep-th/0005204];

[48] L. Alvarez-Gaume and S. R. Wadia, "Gauge theory on a quantum phase space," hep-th/0006219.

[49] N. Seiberg, "A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation," JHEP **0009**, 003 (2000) [hep-th/0008013].

[50] S. R. Das and S. Rey, "Open Wilson lines in noncommutative gauge theory and tomography of holographic dual supergravity," Nucl. Phys. **B590** (2000) 453 [hep-th/0008042].

[51] D. J. Gross, A. Hashimoto and N. Itzhaki, "Observables of non-commutative gauge theories," hep-th/0008075.

[52] A. Dhar and S. R. Wadia, "A note on gauge invariant operators in noncommutative gauge theories and the matrix model," hep-th/0008144, to appear in Phys. Lett. B.

[53] A. Dhar and Y. Kitazawa, "Wilson loops in strongly coupled noncommutative gauge theories," hep-th/0010256.

[54] "Branes from Matrices" T. Banks, N. Seiberg and S. Shenker, "Branes from matrices," Nucl. Phys. **B490** (1997) 91 [hep-th/9612157].

[55] O. Aharony and M. Berkooz, "Membrane dynamics in M(atrix) theory," Nucl. Phys. **B491**, 184 (1997) [hep-th/9611215].

[56] G. Lifschytz and S. D. Mathur, "Supersymmetry and membrane interactions in M(atrix) theory," Nucl. Phys. **B507**, 621 (1997) [hep-th/9612087].

[57] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, "M theory as a matrix model: A conjecture," Phys. Rev. **D55**, 5112 (1997) [hep-th/9610043]. For reviews, see, e.g., [58, 59].

[58] T. Banks, "TASI lectures on matrix theory," hep-th/9911068; "Matrix theory," Nucl. Phys. Proc. Suppl. **67** (1998) 180 [hep-th/9710231].

[59] W. I. Taylor, "The M(atrix) model of M-theory," hep-th/0002016.

[60] R. Dijkgraaf, E. Verlinde and H. Verlinde, "Matrix string theory," Nucl. Phys. **B500** (1997) 43 [hep-th/9703030].

[61] E. Keski-Vakkuri and P. Kraus, "Notes on branes in matrix theory," Nucl. Phys. **B510** (1998) 199 [hep-th/9706196].

[62] W. I. Taylor, "D-brane field theory on compact spaces," Phys. Lett. **B394** (1997) 283 [hep-th/9611042].

[63] L. Susskind, "Another conjecture about M(atrix) theory," hep-th/9704080.

[64] N. Seiberg, "Why is the matrix model correct?," Phys. Rev. Lett. **79** (1997) 3577 [hep-th/9710009].

[65] A. Sen, "D0 branes on T(n) and matrix theory," Adv. Theor. Math. Phys. **2** (1998) 51 [hep-th/9709220].

[66] A. A. Gerasimov and S. L. Shatashvili, "Stringy Higgs Mechanism and the Fate of Open Strings," hep-th/0011009.